■Ngày nhận bài: 10/4/2023 ■Ngày sửa bài: 09/5/2023 ■Ngày chấp nhận đăng: 22/5/2023

Dynamic analysis of cracked plate on Elastic foundation subjected to moving oscillator by finite element method

Phân tích động lực học tấm có vết nứt trên nền đàn hồi chịu tác dụng của tải trọng di động theo phương pháp phần tử hữu hạn

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Received: MM DD, 2023; **Accepted**: MM DD, 2023; **Published**: MM DD, 2023

ABSTRACT

This paper presents the finite element algorithm and results on dynamic response of cracked plate structure on elastic foundation subjected to moving oscillator with a constant velocity and any motion orbit. The response to a single moving oscillator is first investigated and then the effects of crack, velocity, elastic foundation stiffness, and stiffness of the spring of moving oscillator are studied. Results indicate that the number of crack and the foundation stiffness and the velocity of the oscillator moving have significant effects on the dynamic response of the cracked plates.

Keywords: Dynamic; cracked plate; foundation; moving oscillator; finite element method.

TÓM TẮT

Bài báo trình bày thuật toán phần tử hữu hạn và kết quả tính toán đáp ứng động của tấm có vết nứt tựa trên nền đàn hồi chịu tác dụng của tải trọng di động (khối lượng di động và hệ dao động di động) với vận tốc không đổi và quỹ đạo chuyển động bất kỳ. Trong mô hình bài toán, tác giả tính toán đáp ứng động của kết cấu khi chịu tác dụng của một tải trọng di động, sau đó phân tích ảnh hưởng của các thông số vết nứt, độ cứng nền đàn hồi, vận tốc và độ cứng của hệ dao động di động đến đáp ứng động của kết cấu. Các kết quả khảo sát số cho thấy số lượng vết nứt, độ cứng của nền và vận tốc của hệ dao động có ảnh hưởng đáng kể đến đáp ứng động lực học của kết cấu.

Từ khóa: Phân tích động; tấm có vết nứt; nền đàn hồi; tải trọng di động; phương pháp phần tử hữu hạn.

1. INTRODUCTION

In fact, the type of plate structures resting on elastic foundation affected by the moving loads is normally quite a lot in practice, such as pavement, runway pavement, bridge floor, etc. In the process of being subjected to loads and the environment, plate and beam structures on elastic foundation often crack, which results in reduced their stiffness. Therefore, the calculation of dynamic response of plate structures subjected to moving loads have been studied by many scientists. Accordingly, N. T. Chung and L. P. Binh [1] analyzed the cracked beam on the elastic foundation under moving mass by finite element method (FEM). S.R. Mohebpour and P. Malekzadeh [2], P. Malekzadeh *et al.* [3], Qinghua Song *et al.* [4], Qinghua Song *et al.* [5] are presents a finite element model based on the first order shear deformation theory to investigate the dynamic response of laminated composite, FGM plates subjected to a moving oscillator or moving mass. Ahmad Mamandi *et al.* [6] simulated nonlinear dynamic of rectangular plates subjected to accelerated/decelerated moving load by using FEM and ANSYS software. M. H. Huang and D. P. Thambiratnam [7] used FEM to dynamic analysis of plate made by isotropic material on Winkler foundation subjected to a moving concentrated load. A.R. Vosoughi *et al.* [8] analyzed the moderately thick laminated composite plates on elastic foundation subjected to moving load. G.L. Oian, S.N. Gu and J.S. Jiang [9], Marek Krawczuk [10] analyzed the cracked plate subjected to dynamic loads by FEM. N. T. Chung *et al*. [11] presented the finite element algorithm and results of dynamical response of cracked plate subjected to moving oscillator with a constant velocity and any motion orbit. There are many surveys considering the dynamic response of the plate when there is a change in number of cracks and the stiffness of the spring of moving oscillator. Results show that the effect of cracks on the plate's vibration is significan.

2. FINITE ELEMENT SIMULATION AND DOMINANT EQUATIONS

In this research work, an isotropic homogeneous elastic rectangular cracked plate resting on an elastic Winkler foundation under moving oscillator is considered as shown in Figure 1.

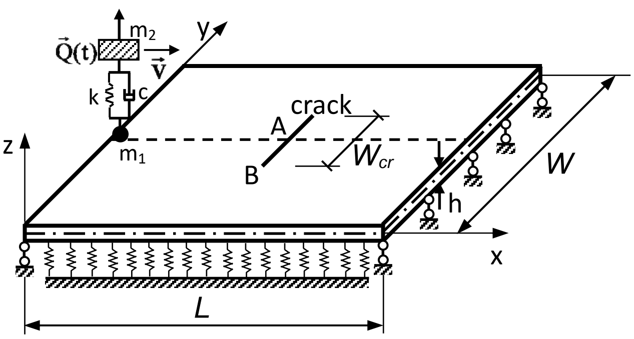

Fig 1. Cracked plate on elastic foundation subjected to moving oscillator

For the finite element model formulation the following assumptions are made: Materials of the system are linear-elastic; cracks are considered penetrating, open and non-propagating while system operate; load and pavement are not speared in the activity duration of system.

2.1. Cracked plate element on elastic foundation subjected to moving oscillator

2.1.1. Cracked plate element on elastic foundation subjected to dynamic loads

Plate is described by bending rectangular four-node elements (Fig.2). Arbitrary point in the element has positions (x,y) in global coordinate and positions (r,s) in local coordinate [15]. Assume that the thickness of plate element *h* is a constant and the conditions of Reissner - Mindlin plate theory are satisfied.

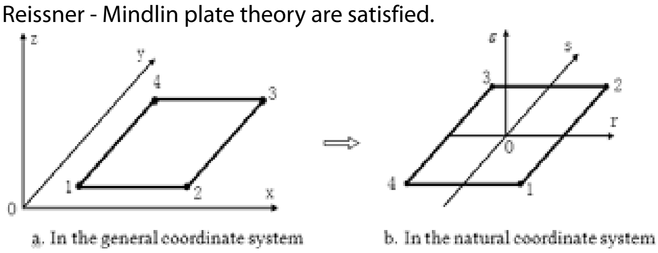

Fig 2. Model of 4-node plate element and the coordinate system

The displacement fields are written as [17]:

$$\begin{cases} u(x,y,z,t)=u_0(x,y,t)+z\theta_y(x,y,t), \\ v(x,y,z,t)=v_0(x,y,t)-z\theta_x(x,y,t), \\ w(x,y,z,t)=w_0(x,y,t), \end{cases} \quad (1)$$

where u_0, v_0, w_0 are the displacements of the midplane and θ_x, θ_y - rotations of normal about respectively the y and x axes.

The strain vector is presented in the form:

$$\{\varepsilon_p\}=\left\{\{\varepsilon_x \quad \varepsilon_y \quad \gamma_{xy}\} \quad \{\gamma_{xz} \quad \gamma_{yz}\}\right\}^T=\left\{\{\varepsilon^b\}^T \quad \{\varepsilon^s\}^T\right\}^T, \quad (2)$$

where

$$\{\varepsilon^b\}=\left\{\frac{\partial u_0}{\partial x} \quad \frac{\partial v_0}{\partial y} \quad \left(\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}\right)\right\}^T+ z\left\{\frac{\partial \theta_y}{\partial x} \quad -\frac{\partial \theta_x}{\partial y} \quad \left(\frac{\partial \theta_y}{\partial x}-\frac{\partial \theta_x}{\partial y}\right)\right\}^T=\{\varepsilon_0\}+z\{\kappa\}, \quad (3)$$

$$\{\varepsilon^s\}=\{\gamma_{xz} \quad \gamma_{yz}\}^T=\left\{\frac{\partial w_0}{\partial x}+\theta_y \quad \frac{\partial w_0}{\partial y}-\theta_x\right\}^T, \quad (4)$$

$$\{\kappa\}=\{k_x \quad k_y \quad k_{xy}\}^T=\left\{\frac{\partial \theta_y}{\partial x} \quad -\frac{\partial \theta_x}{\partial y} \quad \left(\frac{\partial \theta_y}{\partial x}-\frac{\partial \theta_x}{\partial y}\right)\right\}^T. \quad (5)$$

The constitutive equation can be written as

$$\{\sigma\}=\begin{Bmatrix}\{\sigma^b\} \\ \{\sigma^s\}\end{Bmatrix}=\begin{bmatrix}[D^b] & [0] \\ [0] & [D^s]\end{bmatrix}\begin{Bmatrix}\{\varepsilon^b\} \\ \{\gamma^s\}\end{Bmatrix}, \quad (6)$$

where $\{\sigma^b\}$ is stress vector without shear deformation:

$$\{\sigma^b\}=\begin{Bmatrix}\sigma_x \\ \sigma_y \\ \tau_{xy}\end{Bmatrix}=\frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 0,5(1-\nu)\end{bmatrix}\begin{Bmatrix}\varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}\end{Bmatrix}= =[D^b]\{\varepsilon^b\}=[D^b](\{\varepsilon_0\}+z\{\kappa\}), \quad (7)$$

$\{\sigma^s\}$ is stress vector of shear stress:

$$\{\sigma^s\}=\begin{Bmatrix}\tau_{xz} \\ \tau_{yz}\end{Bmatrix}=G\begin{Bmatrix}\gamma_{xz} \\ \gamma_{yz}\end{Bmatrix}=\frac{E}{2(1+\nu)}\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}\begin{Bmatrix}\gamma_{xz} \\ \gamma_{yz}\end{Bmatrix}=[D^s]\{\varepsilon^s\}, \quad (8)$$

with E is elastic modulus of longitudinal deformation, ν is Poisson ratio.

Using Eqs. (7), (8) the components of internal force vector $\{F^{if}\}$ are determined as follows

$$\{M_x \quad M_y \quad M_{xy}\}^T=\int_{-\frac{h}{2}}^{\frac{h}{2}} z\begin{Bmatrix}\sigma_x \\ \sigma_y \\ \tau_{xy}\end{Bmatrix}dz=\frac{h^3}{12}[D^b]\{\kappa\}, \quad (9)$$

$$\{Q_x \quad Q_y\}^T=\int_{-h/2}^{h/2}[D^s]\{\varepsilon^s\}dz=\alpha h[D^s]\{\varepsilon^s\}, \quad (10)$$

So that one obtains

$$\{F^{if}\}=[D^{cs}]\{\varepsilon^{cs}\}, \quad (11)$$

where $[D^{cs}]=\begin{bmatrix}\frac{h^3}{12}[D^b] & [0] \\ [0] & \alpha h[D^s]\end{bmatrix}$ - strain matrix, $\{\varepsilon^{cs}\}=\{k_x \quad k_y \quad k_{xy} \quad \gamma_{xz} \quad \gamma_{yz}\}^T$ is the vector of curvatures and shear strains, α is the shear strain correction factor, usually equal $\alpha = 5/6$.

The displacement of a point of the element represented as [15]:

$$w=\sum_{i=1}^{4}N_i w_i, \ \theta_x=\sum_{i=1}^{4}N_i\theta_{xi}, \ \theta_y=\sum_{i=1}^{4}N_i\theta_{yi}, \quad (12)$$

where w_i, θ_{xi}, θ_{yi} are displacement w, θ_x, θ_y at i[th] node, respectively, N_i are shape functions.

$$\{\varepsilon^{cs}\}_e=[B]\{q\}_e=\sum_{i=1}^{4}[B_i]\{q_i\}, \quad (13)$$

where $[B]_e$ is matrix for internal force determination, $\{q\}_e=\left\{\{q_1\}^T \quad \{q_2\}^T \quad \{q_3\}^T \quad \{q_4\}^T\right\}_e^T$ is vector of node displacement, with $\{q_i\}=\{w_i \quad \theta_{xi} \quad \theta_{yi}\}^T$, (i = 1,2,3,4).

Substituting Eq. (13) into (11) leads to:

$$\{F^{if}\}=\sum_{i=1}^{4}[D^{cs}B_i]\{q_i\}, \quad (14)$$

where $\left[D^{cs}B_i\right]=\left[D^{cs}B_i\right]^b+\left[D^{cs}B_i\right]^s$, (15)

$\left[D^{cs}B_i\right]^b$, $\left[D^{cs}B_i\right]^s$ are matrices corresponding to bending moment and shear force respectively [10].

The dynamic equations of plate element can be derived by using Hamilton's principle [15], [16]:

$$\delta\int_{t_1}^{t_2}\left[T_e-\Pi_e\right]dt=0, \quad (16)$$

where T_e, Π_e are kinetic energy and total potential energy of the element, respectively.

The kinetic energy of the element level is defined as [15]:

$$\Pi_e=\frac{1}{2}\int_{A_e}\left\{F^{\text{if}}\right\}_e^T\left[D^{cs}\right]\left\{F^{\text{if}}\right\}_e dA_e+\frac{1}{2}\int_{A_e}k_f w^2 dA_e-\int_{A_e}\text{w}p dA_e=$$
$$=\frac{1}{2}\{q\}_e^T\left(\left[K_0\right]_e+\left[K_f\right]_e\right)\{q\}_e-\{q\}_e^T\{f\}_e, \quad (17)$$

where

$$\left[K_0\right]_e=\int_{A_e}\left[B\right]^T\left[D^{cs}\right]\left[B\right]dA_e,\qquad \left[K_f\right]_e=\int_{A_e}\left[N\right]^T k_f\left[N\right]dA_e,$$

$$\{f\}_e=\int_{A_e}\left[N\right]^T p dA_e$$

are stiffness matrix (plate element and foundation) and node loading vector of the element, respectively, [N] is mode shape function matrix of element, p is pressure of intensity, k_f is elastic foundation coefficient, w = [N]$\{q\}_e$, A_e is the surface area of the plate elements.

Kinetic energy T_e of element is determined by [15]:

$$T_e=\frac{1}{2}\int_{V_e}\rho\{\dot{u}\}_e^T\{\dot{u}\}_e dV_e$$
$$=\frac{1}{2}\{\dot{q}\}_e^T\left(\int_{V_e}\rho\left[N\right]^T\left[N\right]dV_e\right)\{\dot{q}\}_e=\frac{1}{2}\{\dot{q}\}_e^T\left[M_0\right]_e\{\dot{q}\}_e, \quad (18)$$

where $\left[M_0\right]_e=\int_{V_e}\rho\left[N\right]^T\left[N\right]dV_e$ - mass matrix, ρ - mass density and $\{\dot{q}\}_e$ -velocity vector.

Substituting equations (17) and (18) into equation (16), the dynamic matrix equations of plate element on elastic foundation without damping can be written as:

$$\left[M_0\right]_e\{\ddot{q}\}_e+\left(\left[K_0\right]_e+\left[K_f\right]_e\right)\{q\}_e=\{f\}_e. \quad (19)$$

The cracked plate element stiffness matrix $[K_c]_e$ can be written as [8]:

$$\left[K_c\right]_e=\left[B\right]^T\left[C_f\right]^{-1}\left[B\right], \quad (20)$$

where [B] is the transformation matrix, $\left[C^f\right]=\left[C_0^f\right]+\left[C_1^f\right]$ in which $\left[C_0^f\right]$ is the flexibility matrix of the noncracked element, $\left[C_1^f\right]$ is the flexibility matrix due to the presence of the crack [11], [14].

Now, the dynamic matrix equations of the cracked plate element on elastic foundation subjected to dynamic loads becomes:

$$\left[M_0\right]_e\{\ddot{q}\}_e+\left(\left[K_c\right]_e+\left[K_f\right]_e\right)\{q\}_e=\{f\}_e. \quad (21)$$

2.1.2. Cracked plate element on elastic foundation subjected to moving oscillator

The force of the moving oscillator on the plate at the time t is determined as follows [6]:

$$R=-\left(m_1\frac{d^2w(x,y,t)}{dt^2}+m_2\ddot{u}+\left(m_1+m_2\right)g-Q(t)\right)\Bigg|_{\substack{x=\xi\\ y=\eta}}, \quad (22)$$

where g - acceleration due to gravity, $\ddot{u}$ - acceleration of the mass m_2, $\frac{d^2w(x,y,t)}{dt^2}=\ddot{w}$ - acceleration of the plate at the force setpoint is given by:

$$\frac{d^2w}{dt^2}=\begin{pmatrix}\frac{\partial^2 w}{\partial x^2}\dot{x}^2+\frac{\partial^2 w}{\partial y^2}\dot{y}^2+\frac{\partial^2 w}{\partial t^2}+2\frac{\partial^2 w}{\partial x\partial y}\dot{x}\dot{y}+\\ +2\dot{x}\frac{\partial^2 w}{\partial x\partial t}+2\dot{y}\frac{\partial^2 w}{\partial y\partial t}+\ddot{x}\frac{\partial w}{\partial x}+\ddot{y}\frac{\partial w}{\partial y}\end{pmatrix}. \quad (23)$$

Substituting w = [N]$\{q\}_e$ into Eq. (23), yields:

$$\frac{d^2w}{dt^2}=\begin{pmatrix}\left[N_{xx}\right]\dot{x}^2\{q\}_e+\left[N_{yy}\right]\dot{y}^2\{q\}_e+\left[N\right]\{\ddot{q}\}_e+\\ +2\dot{x}\dot{y}\left[N_{xy}\right]\{q\}_e+2\dot{x}\left[N_x\right]\{\dot{q}\}_e+\\ +2\dot{y}\left[N_y\right]\{\dot{q}\}_e+\ddot{x}\left[N_x\right]\{q\}_e+\ddot{y}\left[N_y\right]\{q\}_e\end{pmatrix}, \quad (24)$$

with $\left[N_x\right]=\frac{\partial\left[N\right]}{\partial x}$, $\left[N_{xx}\right]=\frac{\partial^2\left[N\right]}{\partial x^2}$, $\left[N_{xy}\right]=\frac{\partial^2\left[N\right]}{\partial x\partial y}$, $\left[N_y\right]=\frac{\partial\left[N\right]}{\partial y}$, $\left[N_{yy}\right]=\frac{\partial^2\left[N\right]}{\partial y^2}$, $\dot{x},\dot{y}$ and $\ddot{x},\ddot{y}$ are the velocity and acceleration of the loads along x, y axes, respectively.

Substituting Eq. (24) into (23), the force of the moving oscillator on the plate at the time t can be written as:

$$R=Q(t)-m_1\left[N\right]\{\ddot{q}\}_e-2m_1\left(\dot{x}\left[N_x\right]+\dot{y}\left[N_y\right]\right)\{\dot{q}\}_e$$
$$-m_1\begin{pmatrix}\left[N_{xx}\right]\dot{x}^2+\left[N_{yy}\right]\dot{y}^2+2\dot{x}\dot{y}\left[N_{xy}\right]\\ +\ddot{x}\left[N_x\right]+\ddot{y}\left[N_y\right]\end{pmatrix}\{q\}_e$$
$$-m_2\ddot{u}-\left(m_1+m_2\right)g. \quad (25)$$

Concentrated force (25) is described by the uniformly distributed load as follows [11], [15]:

$$p(x,y,t)=\delta(x-\xi)\delta(y-\eta)R(x,y,t), \quad (26)$$

where δ(.) is the Dirac's delta function.

Substituting equation (25) into equation (26) leads to:

$$p=Q\delta(x-\xi)\delta(y-\eta)-m_1\left[N\right]\delta(x-\xi)\delta(y-\eta)\{\ddot{q}\}_e$$
$$-2m_1\left(\dot{x}\left[N_x\right]+\dot{y}\left[N_y\right]\right)\delta(x-\xi)\delta(y-\eta)\{\dot{q}\}_e$$

$$-m_1\begin{pmatrix}[N_{xx}]\dot{x}^2+[N_{yy}]\dot{y}^2\\ +2\dot{x}\dot{y}[N_{xy}]+\ddot{x}[N_x]+\ddot{y}[N_y]\end{pmatrix}\delta(x-\xi)\delta(y-\eta)\{q\}_e$$
$$-m_2\ddot{u}\delta(x-\xi)\delta(y-\eta)-(m_1+m_2)g\delta(x-\xi)\delta(y-\eta). \quad (27)$$

The element nodal load vector is [15]:

$$\{f\}_e=\int_0^b\int_0^a[N]^T p(x,y,t)\,dxdy=$$
$$=\int_0^b\int_0^a[N]^T\delta(x-\xi)\delta(y-\eta)R(x,y,t)\,dxdy. \quad (28)$$

Substituting equation (27) into equation (28) leads to the nodal load vector:

$$\{f\}_e=\{P\}_e-\left[M_p^{m_1}\right]_e\{\ddot{q}\}_e-\left[M_p^{m_2}\right]_e\ddot{u}$$
$$-\left[C_p\right]_e\{\dot{q}\}_e-\left[K_p\right]_e\{q\}_e, \quad (29)$$

where

$$\{P(t)\}_e=[N(\xi,\eta)]^T(Q-(m_1+m_2)g), \quad (30)$$

$$\left[M_p^{m_1}\right]_e=m_1[N(\xi,\eta)]^T[N(\xi,\eta)], \quad (31)$$

$$\left[M_p^{m_2}\right]_e=m_2[N(\xi,\eta)]^T, \quad (32)$$

$$\left[C_p\right]_e=2m_1[N(\xi,\eta)]^T\left(\dot{x}[N_x(\xi,\eta)]+\dot{y}[N_y(\xi,\eta)]\right), \quad (33)$$

$$\left[K_p\right]_e=m_1[N(\xi,\eta)]^T\begin{pmatrix}[N_{xx}(\xi,\eta)]\dot{x}^2+[N_{yy}(\xi,\eta)]\dot{y}^2+\\ +2\dot{x}\dot{y}[N_{xy}(\xi,\eta)]+\\ +\ddot{x}[N_x(\xi,\eta)]+\ddot{y}[N_y(\xi,\eta)]\end{pmatrix}, \quad (34)$$

Substituting Eq. (29) into (21) leads to the dynamic equation of the cracked plate element on elastic foundation subjected to moving oscillator becomes:

$$\left([M_0]_e+\left[M_p^{m_1}\right]_e\right)\{\ddot{q}\}_e+\left[M_p^{m_2}\right]_e\ddot{u}+\left[C_p\right]_e\{\dot{q}\}_e$$
$$+\left([K_c]_e+[K_p]_e+[K_f]_e\right)\{q\}_e=\{P\}_e. \quad (35)$$

The dynamic equation of mass m_2 can be written as:

$$m_2\ddot{u}+c\dot{u}+ku-c[N]\{\dot{q}\}_e-k[N]\{q\}_e=Q(t), \quad (36)$$

Combining Eqs. (35), (36), we have the dynamic equations of the system consist of cracked plate element on elastic foundation and mass m_2 as follows:

$$\left([M_0]_e+\left[M_p^{m_1}\right]_e\right)\{\ddot{q}\}_e+\left[M_p^{m_2}\right]_e\ddot{u}+\left[C_p\right]_e\{\dot{q}\}_e$$
$$+\left([K_c]_e+[K_p]_e+[K_f]_e\right)\{q\}_e=\{P\}_e$$
$$m_2\ddot{u}+c\dot{u}+ku-c[N]\{\dot{q}\}_e-k[N]\{q\}_e=Q(t), \quad (37)$$

Or

$$\begin{bmatrix}[M_0]_e+\left[M_p^{m_1}\right]_e & \left[M_p^{m_2}\right]_e\\ [0] & m_2\end{bmatrix}\begin{Bmatrix}\{\ddot{q}\}_e\\ \ddot{u}\end{Bmatrix}+\begin{bmatrix}[C_p]_e & [0]\\ -c[N] & c\end{bmatrix}\begin{Bmatrix}\{\dot{q}\}_e\\ \dot{u}\end{Bmatrix}$$
$$+\begin{bmatrix}[K_c]_e+[K_p]_e+[K_f]_e & [0]\\ -k[N] & k\end{bmatrix}\begin{Bmatrix}\{q\}_e\\ u\end{Bmatrix}=\begin{Bmatrix}\{P\}_e\\ Q(t)\end{Bmatrix}. \quad (38)$$

2.2. Governing differential equations for total system

Assembling all elements matrices and nodal force vectors the governing equations of motions of the total system can be derived as:

$$\begin{bmatrix}[M_0]+\left[M_p^{m_1}\right] & \left[M_p^{m_2}\right]\\ [0] & m_2\end{bmatrix}\begin{Bmatrix}\{\ddot{q}\}\\ \ddot{u}\end{Bmatrix}+\begin{bmatrix}[C_p]+[C_R] & [0]\\ -c[N(\xi,\eta)] & c\end{bmatrix}\begin{Bmatrix}\{\dot{q}\}\\ \dot{u}\end{Bmatrix}$$
$$+\begin{bmatrix}[K_0]+[K_c]+[K_p]+[K_f] & [0]\\ -k[N(\xi,\eta)] & k\end{bmatrix}\begin{Bmatrix}\{q\}\\ u\end{Bmatrix}=\begin{Bmatrix}\{P\}\\ Q(t)\end{Bmatrix}, \quad (39)$$

where $[M_0]=\sum_{N_0}[M_0]_e$ is the overall structural mass matrix, $[K_0]=\sum_{N_0-N_c}[K_0]_e$ is the overall structural stiffness matrix with total uncracked elements; $[K_c]=\sum_{N_c}[K_c]_e$ is the overall structural stiffness matrix with total cracked elements, $[K_f]=\sum_{N_0}[K_f]_e$ is the overall foundation stiffness matrix; $\left[M_p^{m_1}\right]=\sum_{Ne_{m1}}\left[M_p^{m_1}\right]_e$, $\left[M_p^{m_2}\right]=\sum_{Ne_{m1}}\left[M_p^{m_2}\right]_e$ are the overall mass matrix due to mass m_1, m_2 moving, respectively; $[C_p]=\sum_{Ne_{m1}}[C_p]_e$ is the overall damping matrix due to mass m_1 moving; $[C_R]=\alpha_R[M_0]+\beta_R([K_0]+[K_c])$ is the overall structural damping matrix [15], [16].

The linear differential equation (39) can be solved by using direct integration Newmark's method. A Matlab program named by Cracked_Plates_Foundation_Moving_2023 (CPFM) was conducted to solve Eq. (39).

3. NUMERICAL ANALYSIS

Consider the rectangular cracked plate resting on elastic Winkler foundation showns as Fig. 1. The data for the plate, foundation, and load for numerical examples treated in this and later sections are given by: L = 100m, W = 10m, h = 0.3m, crack length W_{cr} = 0.5m (it appears in the middle of the plate), E = 3.1×10^{10}N/m², ν = 0.25, ρ = 3200kg/m³, m_1 = 300kg, m_2 = 200kg, k = 1.5×10^5N/m, c = 4.5×10^3Ns/m. The boundary conditions are: free along the longitudinal edges (y = 0, y = 10m) and simply supported along the shorter edges (x = 0, x = 100m).

In this numerical example, the elastic foundation stiffness is set to k_f = 1.0×10^7N/m³. The moving oscillator moves along the centerline (y = 5m), parallel to the x axis with constant amplitude and constant velocity v = 25m/s.

The dynamic response of the plate subjected to moving oscillator (MO) and moving mass M = $m_1 + m_2$ = 500kg (MM) consist of displacement, velocity, and stress are shown on Table 1 and Figs. 3, 4, 5, 6. The maximum values of the above quantities at the point A are shown in Tab. 1.

Table 1. The maximum values are at the point A

Case of load	w_A^{max} [m]	$\dot{w}_A^{max}$ [m/s]	σ_x^{max} [N/m^2]	σ_y^{max} [N/m^2]
MO	0.029	1.194	1.11×10^8	1.05×10^8
MM	0.035	1.372	1.30×10^8	1.36×10^8

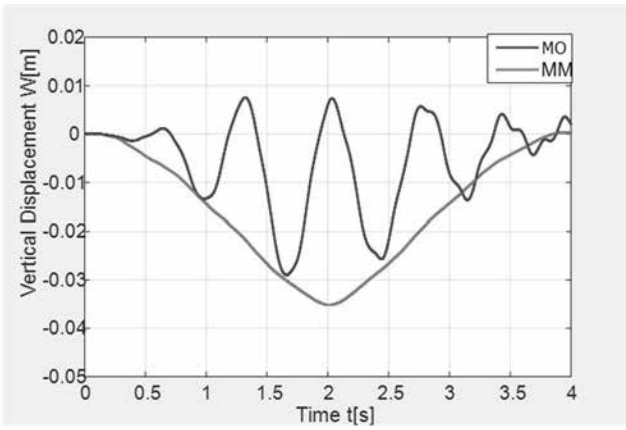

Fig. 3 Dynamic vertical response at point A

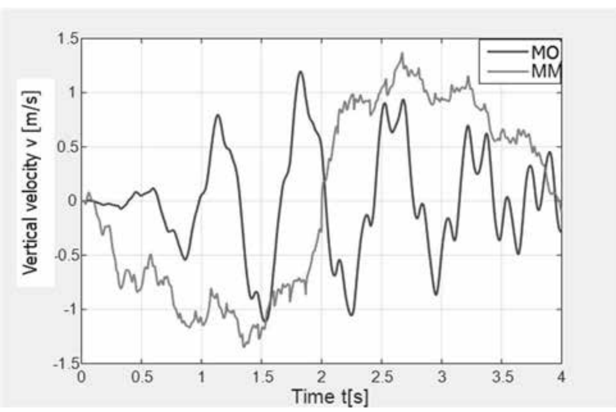

Fig. 4. Vertical velocity response at point A

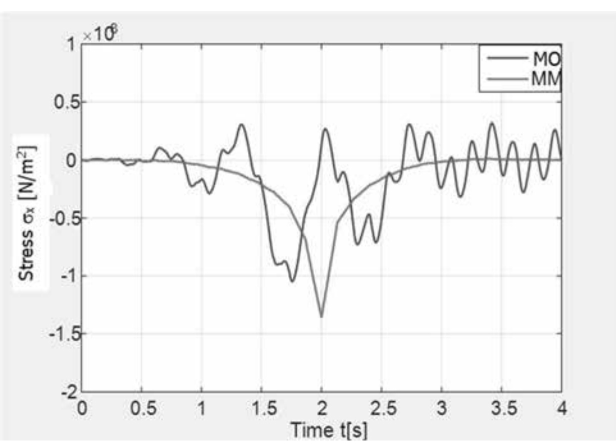

Fig. 5 Stress response σ_x at point A

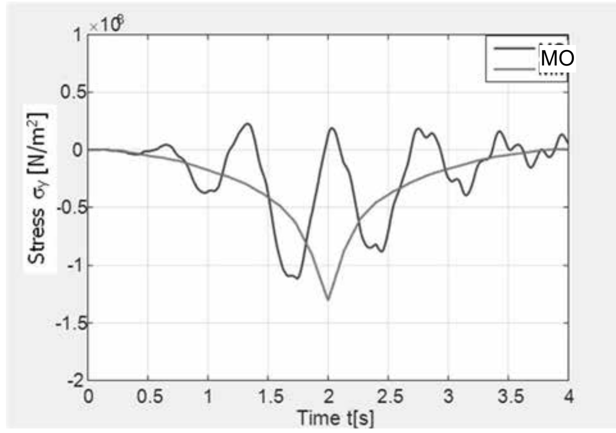

Fig. 6 Stress response σ_y at point A

Comment: Numerical example above show that the dynamic response curves of a cracked plate on elastic foundation subjected to moving oscillator are rough and complex while they are smooth and simple for cracked plate resting on elastic foundation under the corresponding moving mass.

3.1. Effect of the crack length

Evaluating the effect of the crack length to vibration of cracked plate resting on elastic foundation under moving oscillator, numerical caculation with different crack lenghts. Figs. 7, 8, and 9 shows the maximum dynamic deflection at a certain point of the plate (point A) and tip of the crack (point B).

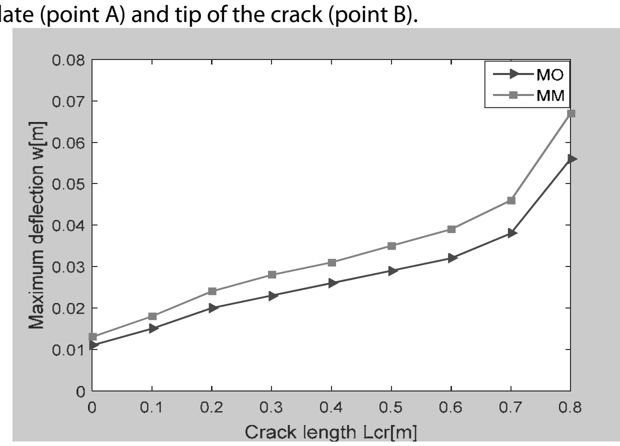

Fig. 7 Maximum dynamic deflection of central point with different crack lengths (v = 25m/s)

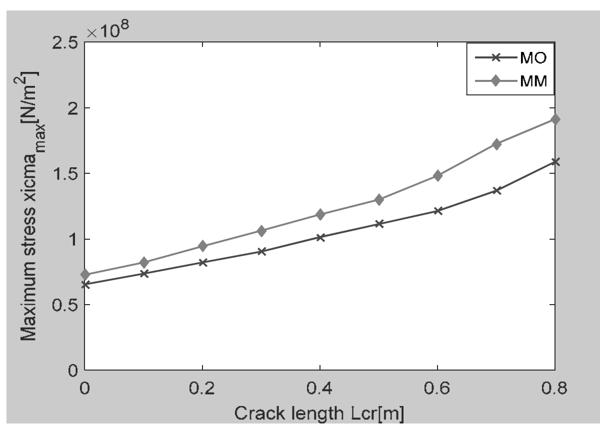

Fig. 8 Maximum dynamic stress σ_y^{max} of central point with different crack lengths (v = 25m/s)

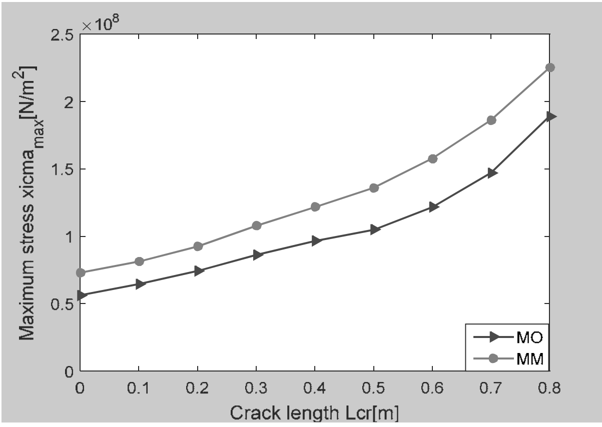

Fig. 9 Maximum dynamic stress σ_x^{max} of tip of the crack with different crack lengths (v = 25m/s)

Comment: Numerical analysis of cracked plates on the elastic foundation under moving oscillator shows that the crack is cause reduces the stiffness of the plate: as the crack length increases, both displacement and stress at the points of the plate increased significantly.

3.2. Effect of the foundation stiffness

When the foundation stiffness k_f increases from 1.0×10^6N/m^3 to 1.0×10^7 N/m^3, Figs. 10, 11, and 12 show the variation of the maximum values of deflection and stress respectively.

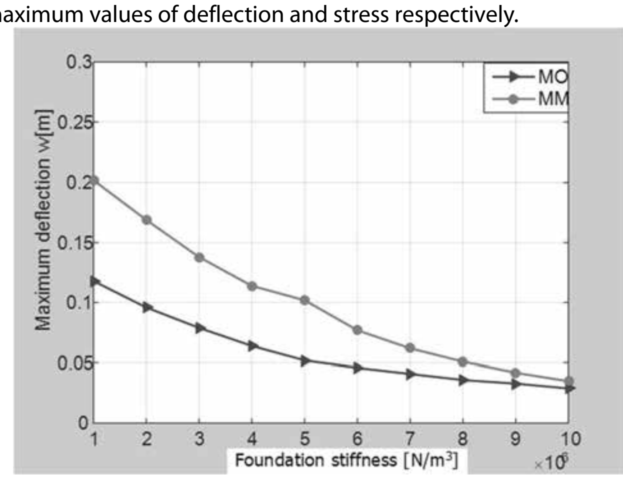

Fig. 10 Maximum dynamic deflection of central point with different foundation stiffness (v = 25m/s)

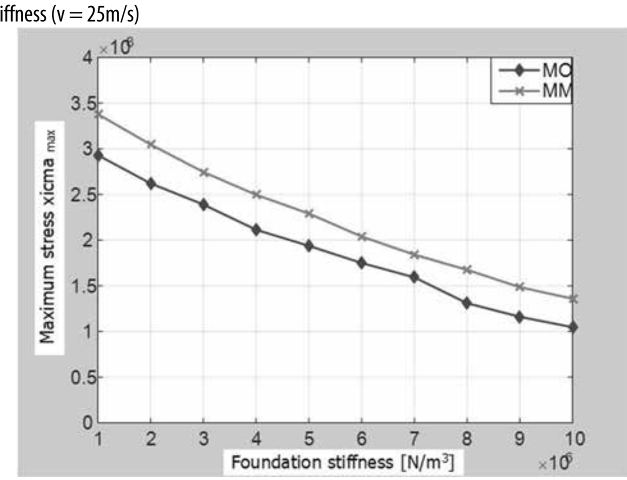

Fig. 11 Maximum dynamic stress σ_y of central point with different foundation stiffness (v = 25m/s)

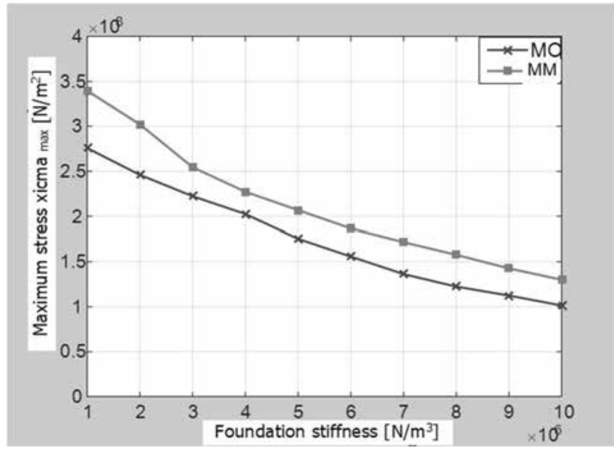

Fig. 12 Maximum dynamic stress σ_x of tip of the crack with different foundation stiffness (v = 25m/s)

Comment: The results show that the elastic foundation stiffness is the factor that significantly reduces the deflection and stress in the plate resting on elastic foundation.

3.3. Effect of spring stiffness of the oscillator

Calculation with the change of spring stiffness value of the moving oscillator from 1.0×10^5N/m to 1.8×10^5N/m. Figs. 15, 16, 17 shows the variation of the maximum dynamic values of the plate.

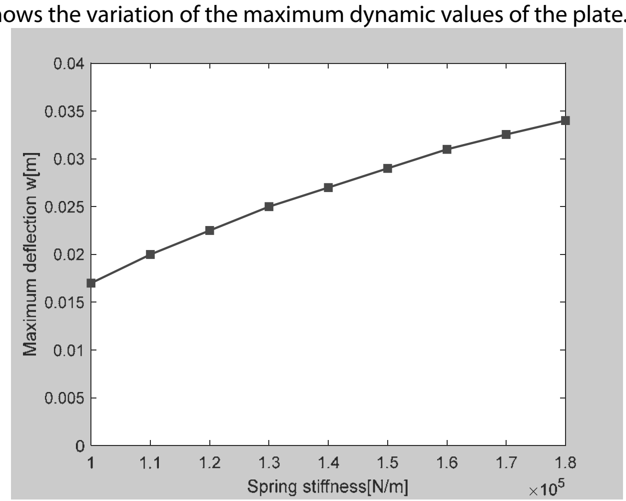

Fig. 13 Maximum dynamic deflection of central point with different spring stiffness (v = 25m/s)

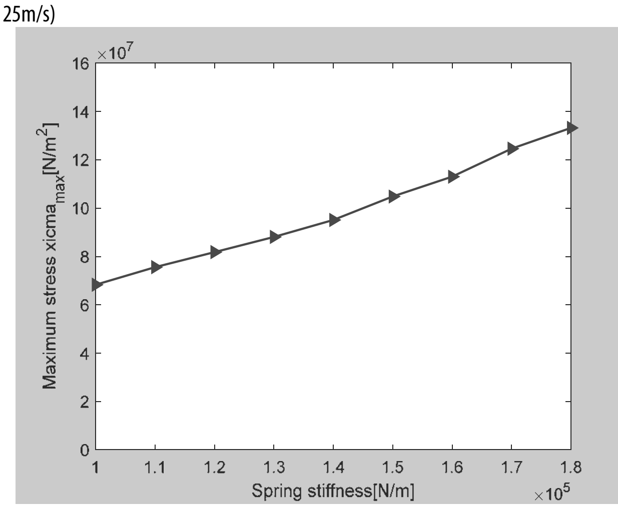

Fig. 14 Maximum dynamic stress σ_y of central point with different spring stiffness (v = 25m/s)

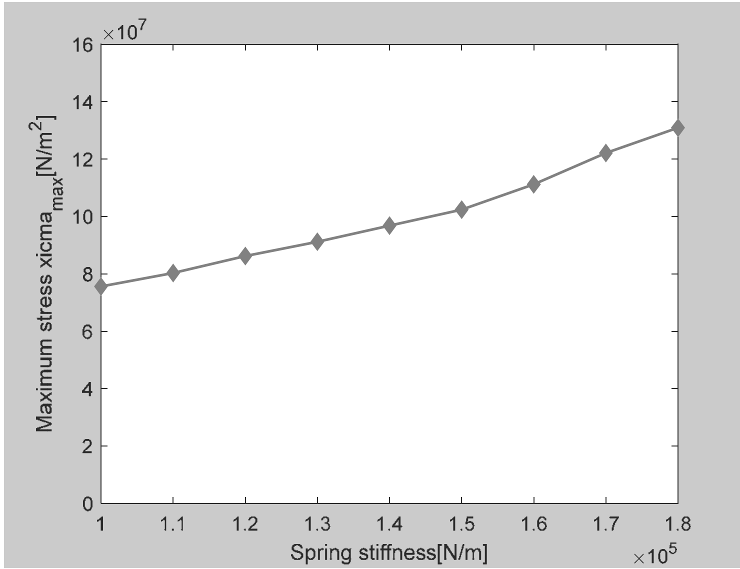

Fig. 15 Maximum dynamic stress σ_x of tip of the crack with different spring stiffness (v = 25m/s)

Comment: When spring stiffness is changed, the oscillation of the system varies considerably. With the parameters of the given plate, the displacement response, and stress at the computed points are the greatest values.

4. CONCLUSIONS

The results numerical analysis of the cracked plate on elastic foundation shows that, with the set of survey parameters, in the case of the cracked plate under the moving mass is more dangerous than moving oscillators operate. However, the problem of texture affected by the oscillation system is a complex problem, requiring more follow-up studies. The response of system consist of cracked plate, moving oscillator, and elastic foundation depends on the interrelation between the frequency of the stimulus and the natural frequency of the system.

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